CISC 372: Parallel Computing

Performance

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Performance: definition

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- how efficiently resources are used to solve a problem

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- resources?
  - memory
  - energy
  - time

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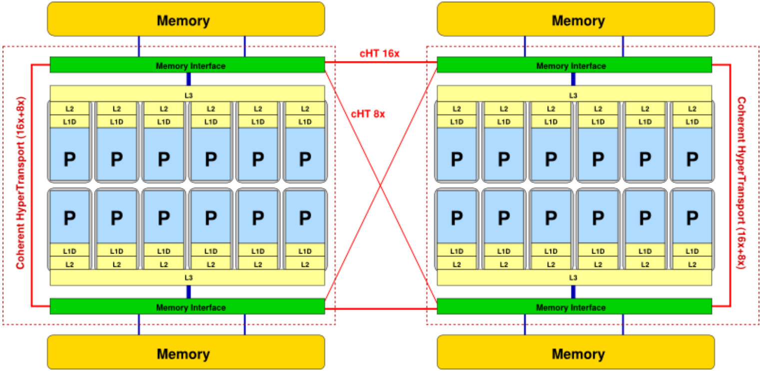
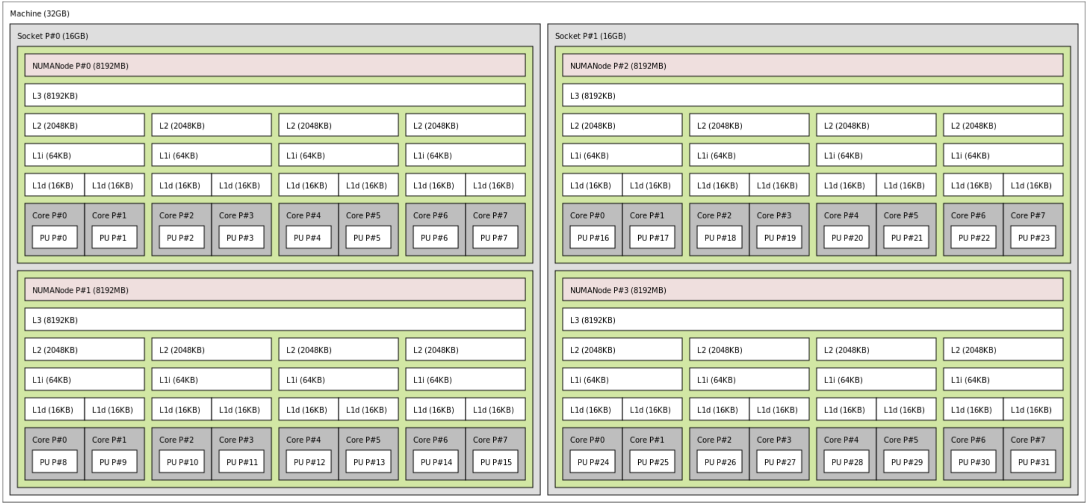


Memory hierarchy: AMD Bulldozer server



https://en.wikipedia.org/wiki/CPU_cache

Memory hierarchy: example: matrix-vector multiplication

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- see `colmaj.c`: a is $N \times N$ array of doubles, $N = 20,000$
  - consider accesses to `a`
  - `a[0][0]`, `a[1][0]`, `a[2][0]`, ...
  - these are separated by $20,000 *$ `sizeof(double)` bytes!
  - each access loads into cache an entire block (cache line) containing the requested location

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- see `rowmaj.c`
  - functionally equivalent to `colmaj.c`
  - `a[0][0]`, `a[0][1]`, `a[0][2]`, ...
  - these are adjacent in memory
  - the first access loads the cache line containing many/all of the subsequent elements

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- see `https://gcc.gnu.org/onlinedocs/gcc/Optimize-Options.html`

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  - do you choose the best possible sequential algorithm that solves the problem?
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  - these are often very different!
  - you must always specify the baseline

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- if seq took 10 seconds and par took 2 seconds, speedup is 5
  - "parallel program is 5x faster than sequential"
  - with those particular inputs and nprocs

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- in best case, speedup may be approximatley linear over some range of nprocs, but never as nprocs $\to \infty$

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  - best possible speedup is 10 :-(
- in general, if inherently sequential fraction of original program is r
  - then speedup $< 1/r$

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- parallelization is more effective with weak scaling than with strong scaling

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  - fix `nx` $= 10^3$; compare sequential diffusion1d vs. parallel diffusion1d with p procs
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- weak scaling: baseline increases with nprocs
  - problem size of sequential program increases with nprocs
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- weak scaling examples
  - for $p > 0$, compare sequential time to sort list of length $10^6 p$ with parallel time using p procs
  - for $p > 0$, compare sequential diffusion1d with `nx` $= 10^3 p$ vs. parallel diffusion1d with p procs
    - note `nxl` $= 10^3$ is held constant as p increases

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- efficiency is "speedup per process"
- Amdahl says that for strong scaling:
  - $\texttt{efficiency} \to 0$ as $\texttt{nprocs} \to \infty$
- for weak scaling, in best case it is possible:
  - $\texttt{efficiency} \to 1$ as $\texttt{nprocs} \to \infty$
- more common: something between 0 and 1

Automating performance experiments

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- see `exp/sat_strong` in public course repo
- a strong scaling experiment of MPI SAT solver

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- a strong scaling experiment of MPI SAT solver
- sat_mpi.c has been altered to
  - print to stdout only the number of processes and time
  - other things are sent to stderr

```
if (rank == 0) {
  const double time = MPI_Wtime() - start_time;

  fprintf(stderr, "Number of solutions = %u.  Time = %lf.\n",
          nsolutions, time);
  fflush(stderr);
  printf("%d %lf\n", nprocs, time);
}
```

Data file generated by SAT performance experiment

- the Makefile executes sat_mpi.exec with 1, 2, 4, 8, 16, 32 procs

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1 42.693483
2 29.942159
4 16.342128
8 9.844605
16 5.327622
32 2.452447
```

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4 16.342128
8 9.844605
16 5.327622
32 2.452447
```

nprocs	time	speedup	efficiency
1	43.75	1.00	1.00
2	30.46	1.44	0.72
4	16.47	2.66	0.66
8	8.68	5.04	0.63
16	4.89	8.95	0.56
32	2.52	17.36	0.54

Graphing data with gnuplot

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- http://www.gnuplot.info
- command: `gnuplot sat_mpi.gnu`

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```
set terminal pdf
set output "sat_mpi.pdf"
set xlabel center "Number of processes"
set ylabel center "time (seconds)"
set xr [0:32]
set yr [0:45]
plot "sat_mpi.dat" using 1:2 title 'MPI' with linespoints
```

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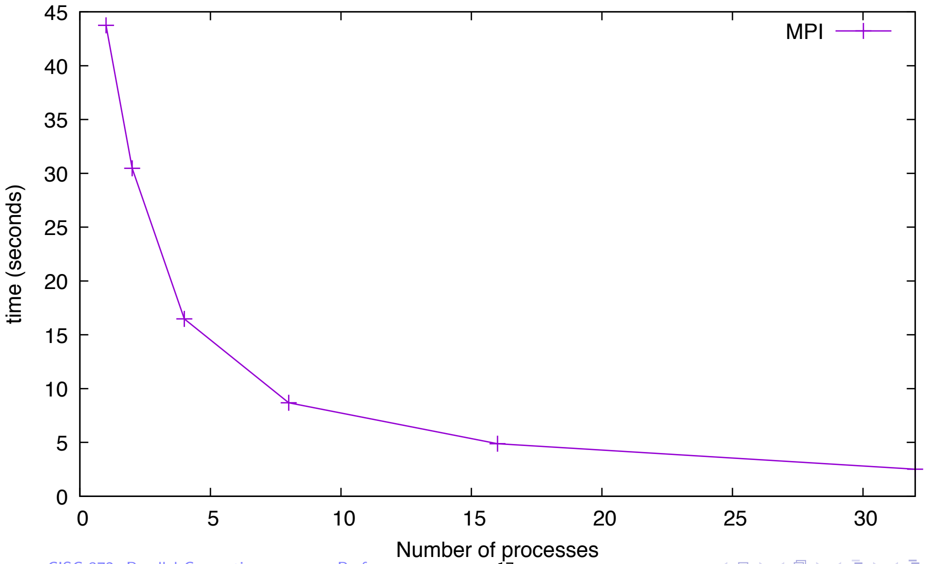
```

- meaning of using 1:2
  - use column 1 of the data file for the x-coordinates
  - use column 2 of the data file for the y-coordinates

PDF file resulting from SAT scaling experiment



Makefile for SAT performance experiment

```
ROOT = ../../
include $(ROOT)/common.mk
NAME = sat_mpi
all: $(NAME).exec
$(NAME).exec: $(NAME).c  Makefile
        $(MPICCC) -o $@ $<
$(NAME).dat: $(NAME).exec
        $(MPIRUN) -n 1 ./$(NAME).exec > $(NAME).dat
        $(MPIRUN) -n 2 ./$(NAME).exec >> $(NAME).dat
        $(MPIRUN) -n 4 ./$(NAME).exec >> $(NAME).dat
        $(MPIRUN) -n 8 ./$(NAME).exec >> $(NAME).dat
        $(MPIRUN) -n 16 ./$(NAME).exec >> $(NAME).dat
        $(MPIRUN) -n 32 ./$(NAME).exec >> $(NAME).dat
graphs:
        gnuplot $(NAME).gnu
.PHONY: all graphs
```

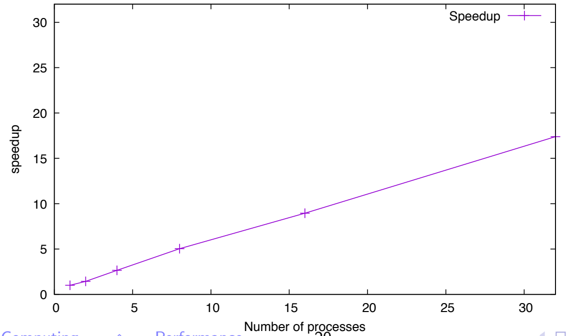
Using gnuplot

Much more is possible...

- graph speedup
- graph efficiency
- graph multiple plots in one picture
  - e.g.: sequential vs. MPI vs. OpenMP

Graphing speedup with gnuplot

```
set output "sat_speedup.pdf"
set xlabel center "Number of processes"
set ylabel center "speedup"
set xr [0:32]
set yr [0:20]
first(x) = ($0 > 0 ? base : base = x)
plot "sat_mpi.dat" using 1:(first($2), base/$2) title 'Speedup' with linespoints
```



Graphing efficiency with gnuplot

```
set output "sat_efficiency.pdf"
set xlabel center "Number of processes"
set ylabel center "efficiency"
set xr [0:32]
set yr [0:1]
first(x) = ($0 > 0 ? base : base = x)
plot "sat_mpi.dat" using 1:(first($2), base/($2*$1)) title 'Efficiency' with linespoints
```