CISC 372: Parallel Computing

Wildcards and Nondeterminism

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  - do not use unless necessary for algorithm
- can use in combination with `MPI_ANY_TAG`

Wildcard receive: example using MPI_ANY_SOURCE: anysource.c

```
#include<stdio.h>
#include<mpi.h>
int main() {
  int message, rank;
  MPI_Init(NULL, NULL);
  MPI_Comm_rank(MPI_COMM_WORLD, &rank);
  if (rank == 0) {
    message = 0; MPI_Send(&message, 1, MPI_INT, 2, 0, MPI_COMM_WORLD);
  } else if (rank == 1)  {
    message = 1; MPI_Send(&message, 1, MPI_INT, 2, 0, MPI_COMM_WORLD);
  } else if (rank == 2) {
    for (int i=0; i<2; i++) {
      MPI_Recv(&message, 1, MPI_INT, MPI_ANY_SOURCE, 0, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
      printf("Proc 2 received: %d\n", message);
    }
  }
  MPI_Finalize();
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    }
  }
  MPI_Finalize();
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```

```
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Proc 2 received: 0
Proc 2 received: 1
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Proc 2 received: 1
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Note:

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Note:

- the receiver can determine if an incoming message matches
  - by examining only the message envelope
- the message data plays no role in determining a match
- the message datatype plays no role in determining a match

Semantics: matching, cont.

- messages within a channel are ordered
  - a receive can only be matched with the oldest matching message in the channel

Semantics: matching, cont.

- messages within a channel are ordered
  - a receive can only be matched with the oldest matching message in the channel
- messages in different channels are not ordered
  - a wildcard (`MPI_ANY_SOURCE`) receive can choose any incoming channel with a matching message
  - and select the oldest matching message from that channel

Message Ordering Example 1

```
Rank 0:
  MPI_Recv(MPI_ANY_SOURCE);
  MPI_Recv(MPI_ANY_SOURCE);

Rank 1:
  MPI_Send(to process 0);

Rank 2:
  MPI_Send(to process 0);
```

Which message gets matched with which receive?

Message Ordering Example 1

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Which message gets matched with which receive?

Answer: either way — no order on messages in different channels

Message Ordering Example 2

```
Rank 0:
  MPI_Recv(MPI_ANY_SOURCE);
  MPI_Recv(MPI_ANY_SOURCE);

Rank 1:
  MPI_Send(to process 0);
  MPI_Send(to process 2);

Rank 2:
  MPI_Recv(from process 1);
  MPI_Send(to process 0);
```

Now process 1 sends its message to 0 before process 2 does.
Which message gets matched with which receive?

Message Ordering Example 2

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Rank 2:
  MPI_Recv(from process 1);
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```

Now process 1 sends its message to 0 before process 2 does.
Which message gets matched with which receive?
Answer: either way — no order on messages in different channels
It doesn't matter that proc 1 sent before proc 2.

Message Ordering Example 3

```
Rank 0:
  MPI_Recv(MPI_ANY_SOURCE);
  MPI_Recv(MPI_ANY_SOURCE);

Rank 1:
  MPI_Send(to process 0);
  MPI_Send(to process 0);
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Which message gets matched with which receive?

Message Ordering Example 3

```
Rank 0:
  MPI_Recv(MPI_ANY_SOURCE);
  MPI_Recv(MPI_ANY_SOURCE);

Rank 1:
  MPI_Send(to process 0);
  MPI_Send(to process 0);
```

Which message gets matched with which receive?

Answer: first send is matched with first receive, second send is matched with second receive

Messages within a channel are ordered

Determinism and nondeterminism

- Programs restricted to
  - deterministic sequential operations
  - only one process performs I/O
  - MPI_Send, MPI_Recv, MPI_Sendrecv, MPI_Init, MPI_Finalize, MPI_Comm_rank, MPI_Comm_size, MPI_ANY_TAG
  - collective operations other than reductions on floating-point numbers
  - but not MPI_ANY_SOURCE

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- are guaranteed to be deterministic
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  - even though the paths from input to output may be differ

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  - collective operations other than reductions on floating-point numbers
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- are guaranteed to be deterministic
  - given the same input twice, same output will be produced
  - even though the paths from input to output may be differ
- But if you add MPI_ANY_SOURCE
  - the program may be nondeterministic
  - given same input twice, two different outputs possible
  - see simplend.c

The need for barrier synchronization

- programs that use `MPI_ANY_SOURCE` sometimes require barrier synchronization!
- this is one of the few times barriers are absolutely needed

Wildcard deadlock example: function f

```
/* Each non-root process sends a message to root.
   Root receives using MPI_ANY_SOURCE.
   This is a perfectly fine deadlock-free function.
 */
void f() {
  if (myrank == 0) {
    MPI_Status status;
    int x;
    for (int i = 1; i < nprocs; i++) {
      MPI_Recv(&x, 1, MPI_INT, MPI_ANY_SOURCE, 0, comm, &status);
      printf("Proc 0: received %d from proc %d\n", x, status.MPI_SOURCE);
    }
  } else {
    MPI_Send(&myrank, 1, MPI_INT, 0, 0, comm);
  }
}
```

Wildcard deadlock example: function g

```
/* Each non-root process sends a message to root.
   Root receives in order of increasing rank.
   This is a perfectly fine deadlock-free function.
 */
void g() {
  if (myrank == 0) {
    MPI_Status status;
    int x;
    for (int i = 1; i < nprocs; i++) {
      MPI_Recv(&x, 1, MPI_INT, i, 0, comm, &status);
      printf("Proc 0: received %d from proc %d\n", x, status.MPI_SOURCE);
    }
  } else {
    MPI_Send(&myrank, 1, MPI_INT, 0, 0, comm);
  }
}
```

Wildcard deadlock example: function main

What happens when I call the two deadlock-free functions in sequence?

```
int main() {
  MPI_Init(NULL, NULL);
  MPI_Comm_size(comm, &nprocs);
  MPI_Comm_rank(comm, &myrank);
  f();
  g();
  MPI_Finalize();
}
```

Wildcard deadlock example: function `main`

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```

```
mpiexec -n 4 ./wcdl.exec
Proc 0: received 1 from proc 1
Proc 0: received 1 from proc 1
Proc 0: received 2 from proc 2
^C[mpiexec@basie.local]
```

Deadlock!

Wildcard deadlock example: what happened?

1. proc 1 in function `f` sent message to proc 0
2. proc 1 in function `g` sent message to proc 0
3. proc 1 terminates
4. proc 2 in function `f` sent message to proc 0
5. proc 0 in function `f` received message from proc 1 at `MPI_ANY_SOURCE`
6. proc 0 in function `f` received message from proc 1 at `MPI_ANY_SOURCE`
7. proc 0 in function `f` received message from proc 2 at `MPI_ANY_SOURCE`
8. proc 0 in function `g` waits for message from proc 1

A message from proc 1 in `g` was received by proc 0 in `f`.

- not what the programmer intended

Wildcard deadlock example: a solution

- place a barrier between f and g
- no one will be able to enter g until everyone has completed f

```
int main() {
  MPI_Init(NULL, NULL);
  MPI_Comm_size(comm, &nprocs);
  MPI_Comm_rank(comm, &myrank);
  f();
  MPI_Barrier(comm);
  g();
  MPI_Finalize();
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}
```

```
mpiexec -n 4 ./wcdl.exec
Proc 0: received 3 from proc 3
Proc 0: received 1 from proc 1
Proc 0: received 2 from proc 2
Proc 0: received 1 from proc 1
Proc 0: received 2 from proc 2
Proc 0: received 3 from proc 3
```

Load Balancing

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  - diffusion, π, sat
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- load balancing crucial to performance
- some problems can be broken up in a predictable way
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  - each process does (roughly) same amount of work
- for other problems this is difficult
  - no way to predict how long a task will take
  - many numerical algorithms require iterating until convergence
  - example: numerical integration
  - heterogenous hardware: processors running at different speeds
  - cyclic distributions are not always going to solve this problem

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  3. processes result and sends new task to that worker
  4. if no tasks remain, sends termination signal to worker instead
  5. when all results have been returned and termination signals sent, finished

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- worker
  1. waits for task from manager
  2. solves the task and sends result to manager
  3. repeat until termination signal received

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- manager must use some nondeterminisitic MPI construct, e.g.
  - `MPI_ANY_SOURCE`
  - `MPI_Waitany`
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- a correct program should return same result independent of these choices

Manager-worker example: matrix-matrix multiplication

A: $N \times L$ B: $L \times M$ C: $N \times M$

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Example: $N = 4$, $L = 3$, $M = 2$

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  - multiply row i of A by B to get row i of C $(i = 0, \ldots N - 1)$

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$$\begin{bmatrix} A[0] \\ A[1] \\ A[2] \\ A[3] \end{bmatrix} \times B = \begin{bmatrix} A[0] \times B \\ A[1] \times B \\ A[2] \times B \\ A[3] \times B \end{bmatrix}$$

Matrix-matrix multiplication: `matmat.c` `matmat_mpi.c`

- sequential solution: `matmat.c`
- parallel solution: `matmat_mpi.c`
  - uses manager-worker pattern
  - a task: one row of A times B to get one row of C

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  - could run the code twice and get two different answers
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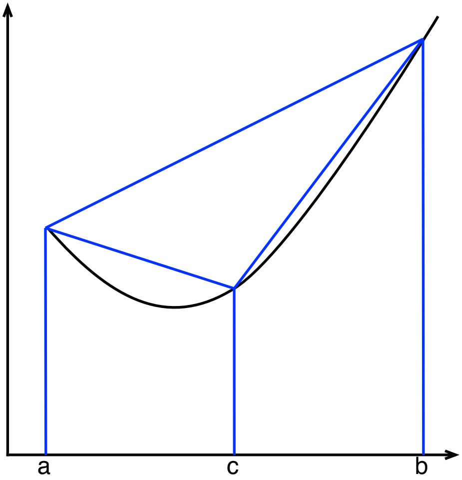
- to parallelize sequential programs with floating point operations, for greatest assurance . . .
  - floating point operations should be identical in both programs
  - if one computes `a+(b+c)` the other must compute `a+(b+c)`, not `(a+b)+c`
  - then they will get exact same result in all cases

Example: Numerical Integration with Trapezoid Rule



- compute area A of large trapezoid
- divide interval in half
- compute areas A_l and A_r of left and right trapezoids
- compare $A_l + A_r$ with A
- if difference is "sufficiently small" return $A_l + A_r$
- else call recursively on left and right subintervals and return sum

See `integral.c`

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- balanced static partitioning of work not possible
- manager-worker pattern called for
  - break up $[a, b]$ into subintervals in exact same way as sequential
  - many more subintervals than processes (e.g., $100\times$)
  - task: compute integral over one subinterval
  - give each worker one task, ...
  - manager sums results in exact same order as sequential

See `integral_mpi.c`